

Q1.

- (a) Simplify $f(x, y) = x(x + y + 2) + 2 - x$
- (b) Work out $f(x, 2)$
- (c) For which x does $f(x, 2) = 0$ by completing the square.
- (d) Plot $y = f(x, 2)$

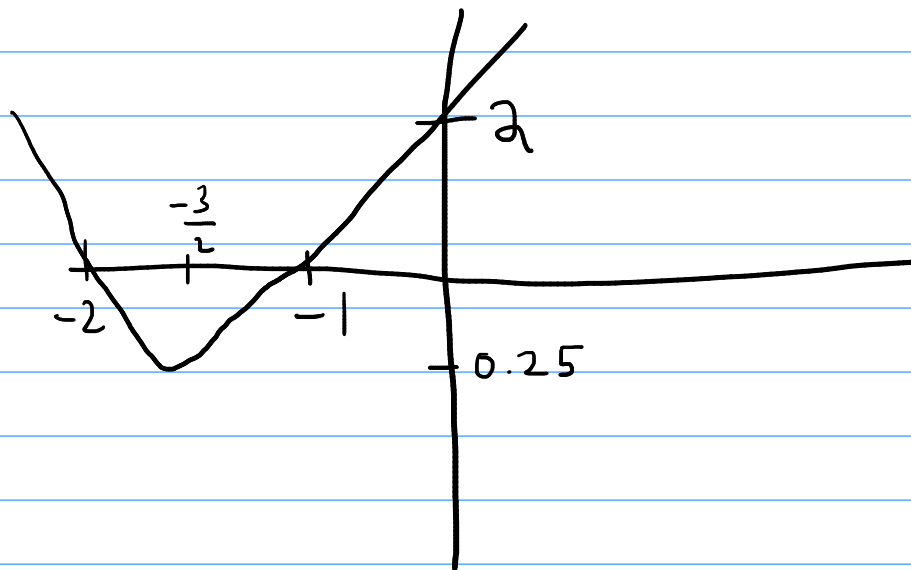
$$\begin{aligned} \text{a) } f(x, y) &= x^2 + xy + 2x - x + 2 \\ &= x^2 + xy + x + 2 \end{aligned}$$

$$\text{b) } f(x, 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

$$\begin{aligned} \text{c) } \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2 &= 0 \\ \left(x + \frac{3}{2}\right)^2 &= \frac{9}{4} - \frac{8}{4} \\ &= \frac{1}{4} \end{aligned}$$

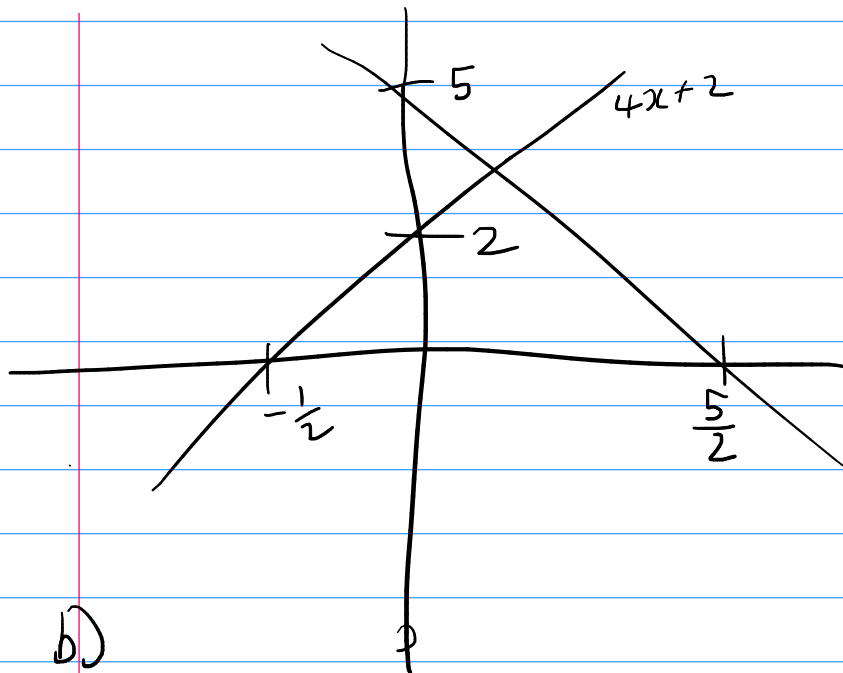
$$\text{Then } x + \frac{3}{2} = \pm \frac{1}{2}$$

$$x = -2 \quad \text{or} \quad x = -1$$



Q2.

- (a) Plot the two line graphs $y = 4x + 2$ and $y = -2x + 5$
- (b) Make sure the graph indicates where the lines cross the x and y axis
- (c) Using your graph approximate where the lines intersect
- (d) Calculate exactly where the lines intersect.
- (e) calculate the midpoint of the line from the origin to the intersection point



b)

c) $4x + 2 = -2x + 5$

$$6x = 3$$

$$x = \frac{1}{2}$$

so $y = 4x + 2$ so $y = 4\left(\frac{1}{2}\right) + 2 = 2 + 2 = 4$

so the intersection point is $\left(\frac{1}{2}, 4\right)$.

d) Want to calculate midpoint of line from $(0,0)$

to $\left(\frac{1}{2}, 4\right)$ so $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ and $4 \times \frac{1}{2} = 2$ so

the midpoint is $\left(\frac{1}{4}, 2\right)$.

①

②

Q3. Solve the simultaneous equations $m + n = 5$ and $4m + 5n = 23$

$$\textcircled{2} - 4\textcircled{1} : (4m + 5n) - 4(m + n) = 23 - 20$$
$$n = 3$$

$$\text{So } m = 5 - n \text{ therefore } m = 5 - 3 = 2$$

Q4. Solve $\frac{1}{x+3} - \frac{1}{x+1} = 2$

$$\frac{(x+1) - (x+3)}{(x+1)(x+3)} = 2$$

$$\text{So } -2 = 2(x+1)(x+3)$$

$$\text{Now, } -1 = (x+1)(x+3)$$

$$x^2 + 4x + 4 = 0 \text{ therefore}$$

$$(x+2)^2 = 0$$

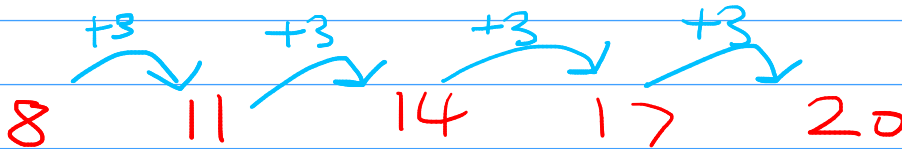
$$\text{So } x = -2.$$

Q5.

The following table depicts the number of visitors to my website in it's first five days of going up.

Day	1	2	3	4	5
Visitors	8	11	14	17	20

- (a) Work out the n th term of this sequence
- (b) Work out the number of viewers on the 10th day
- (c) Work out the number of days for the number of viewers to go above 100.



a) $3n + 5$

b) $3(10) + 5 = 35$

c)

For what n is $3n + 5 > 100$

so $3n > 95$

$n = 31 \quad 3(31) = 93$

$n = 32 \quad 3(32) = 96$

therefore $n = 32$

Q6.

Let $f = x^3 - 5x + 3$,

- (a) Show without explicitly calculating it that f has a root between 0 and 1
- (b) Let $x^3 - 5x + 3 = 0$ then show that $x = \frac{x^3+3}{5}$
- (c) Starting with $x_0 = 0$ use the iterative formula twice $x_{n+1} = \frac{x_n^3+3}{5}$ to find an approximation for the root.

a) let $x=0$, then $f(x) = f(0) = 3$
let $x=1$, then $f(x) = f(1) = -1$
then because f is continuous there must exist an y between 0 and 1 such that $f(y) = 0$.

b) $x^3 - 5x + 3 = 0$ so

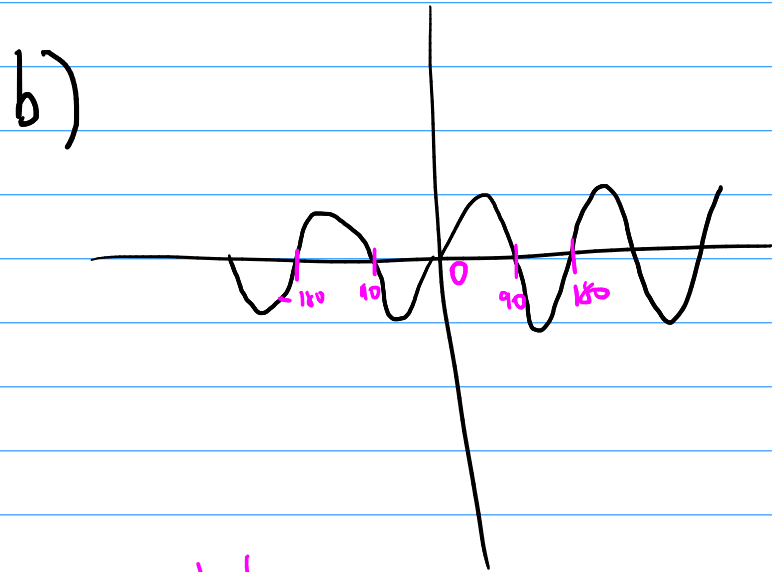
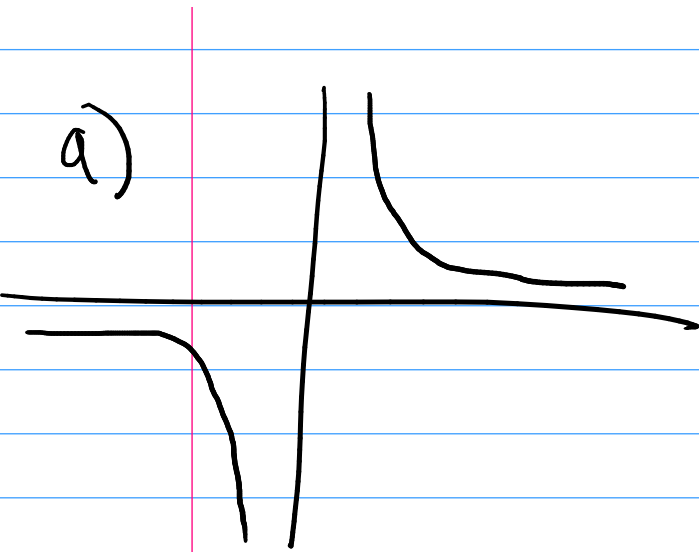
$5x = x^3 + 3$ then

$$x = \frac{x^3 + 3}{5}$$

$$x_0 = 0 \quad x_1 = \frac{0+3}{5} \quad x_2 = \frac{\left(\frac{3}{5}\right)^3 + 3}{5} \\ = \frac{3}{5} \\ = 0.64$$

Challenge

1. Plot $y = \frac{3}{x}$
2. Plot $y = \sin(2x)$
3. Find the roots of $y = \sin(x) + \frac{3}{x}$
4. Try to plot $y = \sin(x) + \frac{3}{x}$
5. Check in Desmos: <https://www.desmos.com/calculator>



c) this is impossible!!

d) AS $x \rightarrow \pm\infty$ $y = \sin(x) + \frac{1}{x}$

starts to look like $\sin(x)$ since
 $\frac{1}{x} \rightarrow 0$

